Lesson 004
Measures of Variability

September 18, 2023

	Student 1	Student 2	Student 3
Course #1	80	70	60
Course #2	80	75	60
Course #3	80	85	100
Course #4	80	90	100
Mean / Median	**80 / 80**	**80 / 80**	**80 / 80**

Variability

When some data are more *spread out* than others, we say that they have higher **variability**.

There is less concentration around the measures of location.

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  - The square root of the sample variance is called the **standard deviation**, $s = \sqrt{s^2}$.

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Range / Variance	**0 / 0**	**20 / 62.5**	**40 / 400**

Suppose that the following 5 values are observed: {7, 8, 5, 5, 7}. What is the range of the data?

0

Range = $8 - 7 = 1$

0%

Range = $7 - 7 = 0$

0%

Range = $7 - 5 = 2$

0%

Range = $8 - 5 = 3$

0%

Suppose that the following 5 values are observed: {7, 8, 8, 5, 7}. What is the variance of the data?

Variance = $\frac{(7-7)^2+(8-7)^2+(8-7)^2+(5-7)^2+(7-7)^2}{5} = 1.2$

0%

Variance = $\frac{(7-7)+(8-7)+(8-7)+(5-7)+(7-7)}{5} = 0$

0%

Variance = $\frac{7^2+8^2+8^2+5^2+7^2}{5} = 50.2$

0%

Variance = $\frac{7^2+8^2+8^2+5^2+7^2}{5} - 7 = 43.2$

0%

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  - The standard deviation will be multiplied by $|c|$.
  - This can be useful for unit conversions.

The variance of temperature, measured in Celsius, was observed to be 25. These temperatures are converted to Kelvin, by adding 273.15. What is the variance and standard deviation of the new data?

Variance is $25 + 273.15 = 298.15$ and standard deviation is $\sqrt{298.15} = 17.267$.

0%

Variance is $25 \times 273.15 = 6828.75$ and standard deviation is $\sqrt{6828.75} = 82.636$.

0%

Variance is 25 and standard deviation is 5.

0%

There is not enough information given to answer either.

0%

The variance of temperature, measured in Celsius, was observed to be 25. These temperatures are converted to Fahrenheit, by first multiplying by 1.8 then adding 32. What is the variance and standard deviation of the new data?

Variance is $1.8 \times 25 + 32 = 77$ and standard deviation is $\sqrt{77} = 8.775$.

0%

Variance is $1.8^2 \times 25 = 81$ and standard deviation is $\sqrt{81} = 9$.

0%

Variance is 25 and standard deviation is 5.

0%

Variance is $1.8 \times 25 = 45$ and standard deviation is $\sqrt{45} = 6.708$.

0%

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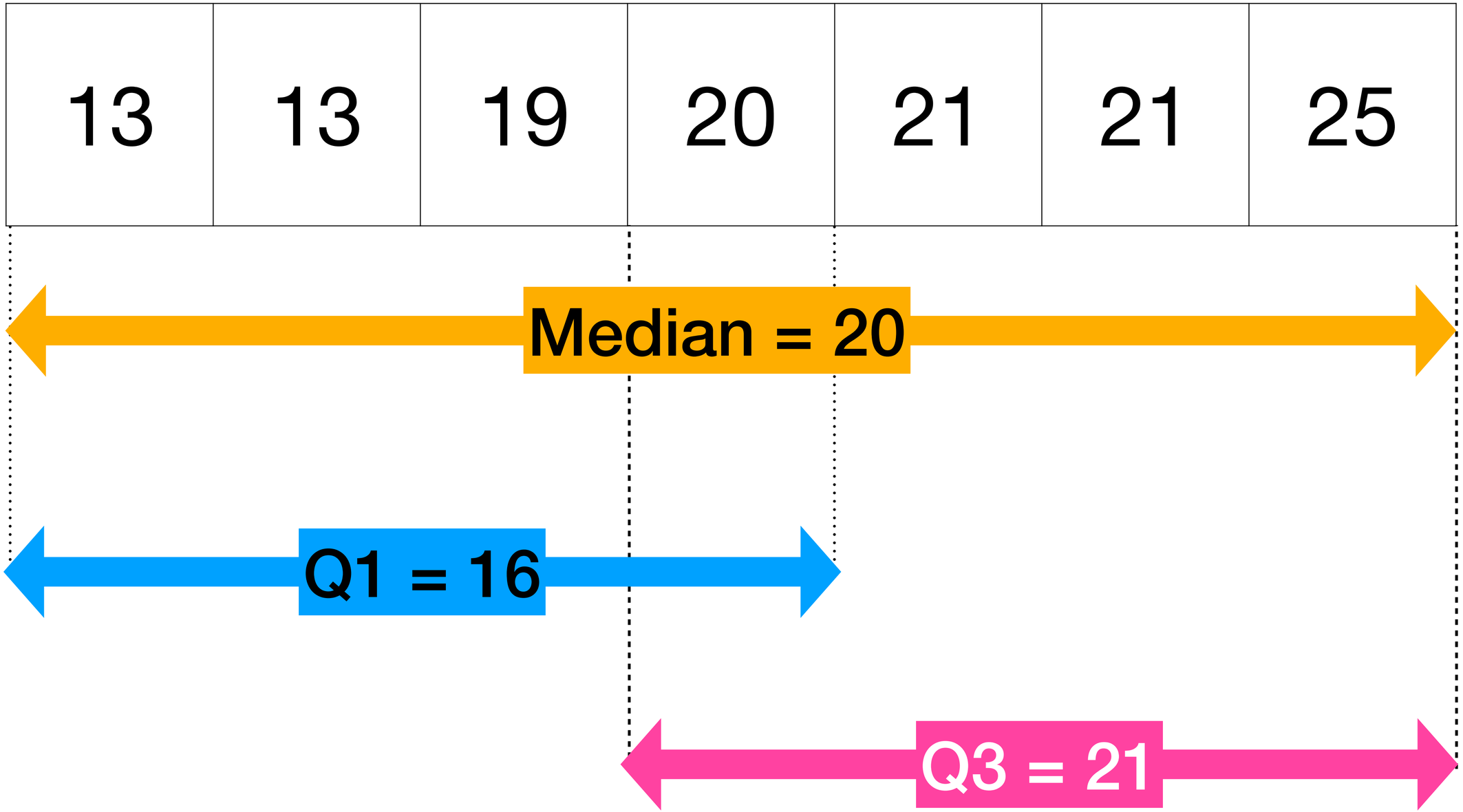
- We call the 25th percentile Q1, and the 75th percentile Q3.
  - This stands for **q**uartile 1 and 3.
  - These can be computed as the median of the lower and upper half of the data.


13
13
19
20
21
21
25
Median = 20
Q1 = 16
Q3 = 21

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  - We can display the five number summary using a **box plot**

13	13	19	20	21	21	25

Min	Q1	Median	Q3	Max	IQR
13	16	20	21	25	5

Consider the data given by {1, 1, 2, 2, 3, 5}. What is Q1, Q3, and the IQR?

$\mathrm{Q1} = 1; \mathrm{Q3} = 3; \mathrm{IQR} = 4$

0%

$\mathrm{Q1} = 1.5; \mathrm{Q3} = 2.5; \mathrm{IQR} = 1$

0%

$\mathrm{Q1} = 1; \mathrm{Q3} = 3; \mathrm{IQR} = 2$

0%

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0%